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A study of the average distribution of pollution around Staythorpe

by

P. J. Meade and F. Pasquill

(Meteorological Office)

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

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Summary

An examination has been made of the average distribution of sulphur pollution around Staythorpe power station in relation to sulphur output and wind statistics during the period November, 1951 to September, 1954.

The data for winter and summer half-years separately are shown to support a relation, which follows from conventional treatment of diffusion from a point source, of the form

$$\chi = a + b\,\frac{f(\theta)\,S}{\bar{u}}$$

In this relation χ is the observed pollution for the half-year, at a distance of 1,500 yards and averaged over an octant about the power station, $f(\theta)$ is the percentage frequency of occurrence of wind directions in the corresponding octant, $\bar{u}$ is the corresponding mean wind speed, and S is the average monthly output of sulphur dioxide during the half-year.

The quantity " a ", logically a "background" pollution, has values which are close to the winter and summer pollution existing immediately before the power station was commissioned. The magnitude of the coefficient " b " is shown to be consistent with previous ideas and other recent data on the vertical diffusion from an elevated point source.

1. Introduction

Over a number of years surveys of sulphur dioxide pollution around power stations have been carried out by the Central Electricity Authority and by the Fuel Research Station of the Department of Scientific and Industrial Research. Analysis of the observational data thus obtained has hitherto generally failed to reveal any significant correlation between the amount of coal consumed at a station and the average sulphur dioxide pollution as measured at a number of sites surrounding the station. It has proved difficult to account for this absence of correlation, and a number of possibilities, mainly concerned with variations in meteorological conditions from one period to another, have been suggested.

With the object of studying in more detail the meteorological effects involved and at the invitation of the Atmospheric Pollution Research Committee, the authors have examined the reports of pollution surveys round a number of power stations. The analysis of observations described in these reports consisted mainly of a direct comparison between coal consumption or sulphur dioxide output and the average values of the pollution measured at all the observing sites around each power station. It appeared, however, that such a procedure was not sufficiently detailed to permit any correlation, if one existed, to emerge.

/It

It was decided therefore to examine the observations of sulphur dioxide pollution as measured at the individual sites around a power station. In this manner the pattern of pollution instead of its overall average became the main subject of investigation and the objective was to ascertain whether this pattern could be explained in terms of the total sulphur dioxide output and the local distribution of wind speed and wind direction. An investigation on these lines concerned with one power station, that at Staythorpe in Lincolnshire, is described in the present paper. Staythorpe is a country district, remote from such complicating factors as built-up areas and industrial plants, and its power station was therefore considered the most appropriate for a first or pilot investigation of this kind. Observations made before the power station was commissioned had shown a uniform distribution of sulphur dioxide pollution in the area. In the following sections the results obtained from an analysis of the Staythorpe data for the three winters and three summers from November, 1951 to September, 1954 are described.

2. The data available

In the absence of wind records for Staythorpe itself, those for the nearest meteorological station, Swinderby, were used. Staythorpe lies about 10 miles to the southwest of Swinderby and it seems highly probable that the wind conditions at the two places are closely similar.

Figure 1 shows the average SO_2 pollution around Staythorpe for each season from the summer of 1948 to the summer of 1954 and also the total coal consumption each season from the time of commissioning the power station in the summer of 1950. Thus the data in this figure, although going back well beyond the period with which the present investigation is concerned, show usefully the seasonal trend in average SO_2 pollution both before and after the power station came into operation. It will be noted from Figure 1 that each season consists of five months, the summer extending from May to September and the winter from November to March.

Table 1, which is supplementary to Figure 1, gives for each season the coal consumption and sulphur dioxide emission expressed in thousands of tons per month and in tons per month respectively.

Table 1

Fuel Consumption and Sulphur Dioxide Emission - Staythorpe

Season	Fuel consumed thousand tons/month	SO_2 emission tons/month
Winter 1951-52	25.2	273
Summer 1952	34.0	536
Winter 1952-53	49.9	881
Summer 1953	38.2	706
Winter 1953-54	51.6	1050
Summer 1954	55.3	1210

At Staythorpe concentrations of sulphur dioxide in the air are measured (Meetham, 1956) by the "lead peroxide" method. A glass or porcelain cylinder is coated with lead peroxide paste and exposed in a louvered box which ensures protection from rain while permitting the free access of air. The paste absorbs sulphur dioxide from the air to form lead sulphate. After exposure for one calendar month the cylinder is removed and analysis of the paste then gives the amount of sulphur dioxide absorbed. This quantity is expressed as milligrammes of SO_3 per day per 100 cm^2.

In Figure 2 the 14 observing sites round the power station at Staythorpe are indicated by means of small circles. The figures within the circles are the measured concentrations (in mg of SO_3/day 100 cm^2) for the winter period November, 1951 to March, 1952. Contours have been drawn to represent

/concentrations

concentrations of 0.9 and 1.0 in these units. Corresponding diagrams for the other five seasons are not reproduced but estimated concentrations derived from them are given in Tables 4 and 5. In Figure 1 the average values of pollution plotted for each season are the means of the observations recorded at all 14 sites.

The percentage frequency of winds in each octant and the weighted mean speed for each octant for the three winters and three summers respectively at Swinderby are shown in Tables 2 and 3 below. The raw data showed for each octant the total times during which the wind speed fell within the ranges 1-3, 4-6, 7-10, etc., knots and for reasons which will appear later the mean wind speed was derived from the reciprocal of the speed in the middle of each range weighted according to the given time. After the mean speeds had been calculated, the percentage of calms in a season was distributed uniformly among the eight octants which in Tables 2 and 3 are shown in clockwise order from the north.

Table 2

Swinderby Wind Distribution - Winter Seasons

Octant	Winter 1951-52		Winter 1952-53		Winter 1953-54	
	percentage frequency	mean speed (kt)	percentage frequency	mean speed (kt)	percentage frequency	mean speed (kt)
0 - 45°	4.2	5.4	10.4	7.0	6.9	6.9
45 - 90°	5.1	5.4	7.0	6.1	11.0	6.7
90 - 135°	7.4	5.4	4.1	5.1	11.2	7.6
135 - 180°	13.5	7.0	7.8	4.5	17.3	8.1
180 - 225°	21.7	9.7	24.4	7.5	26.7	9.0
225 - 270°	23.9	11.3	15.2	8.4	11.6	9.7
270 - 315°	12.7	9.5	8.1	8.3	5.8	8.8
315 - 360°	11.5	7.8	23.0	8.5	9.5	8.0

Table 3

Swinderby Wind Distribution - Summer Seasons

Octant	Summer 1952		Summer 1953		Summer 1954	
	percentage frequency	mean speed (kt)	percentage frequency	mean speed (kt)	percentage frequency	mean speed (kt)
0 - 45°	9.2	6.9	9.4	6.9	8.7	8.0
45 - 90°	6.8	5.7	6.2	5.8	2.3	4.6
90 - 135°	4.5	3.1	6.4	5.5	2.7	5.0
135 - 180°	8.2	5.3	10.6	7.3	11.0	7.2
180 - 225°	17.7	7.1	25.0	9.3	25.3	9.6
225 - 270°	24.1	9.3	21.2	9.6	22.0	10.0
270 - 315°	16.8	8.3	11.5	9.4	13.5	8.9
315 - 360°	12.7	7.1	9.7	6.6	14.5	8.3

3. Analysis procedure

In the present analysis we shall be concerned with the ground level concentration over a circle of radius x from a point source of strength Q units per second at height h. If the cloud is instantaneously narrow in the lateral (crosswind) direction, a condition which will normally hold, the integrated concentration around the circle may be taken to be equal to the

/concentration

concentration at distance x from an infinite crosswind line source of strength Q units/cm sec. at the same height. It then follows that the average concentration in any octant of the circle must be of the form

$$\frac{f(\theta)}{12.5}\ \frac{1}{2\pi x}\ g(h,x)\ \frac{Q}{\bar{u}} \qquad (1)$$

where: $\bar{u}$ = mean wind speed assumed constant with height
$f(\theta)$ = percentage frequency of wind directions in the corresponding octant
$g(h,x)$ = function of the vertical shape and size of the cloud. If, for example, the vertical distribution is Gaussian with standard deviation σ, the wind velocity is constant with height and the cloud is simply reflected from the ground -

$$g(h,x) = \frac{2}{\sqrt{2\pi}\,\sigma} \exp\left(-h^2/2\sigma^2\right) \qquad (2)$$

(This expression follows immediately from a crosswind integration of the point source formula deduced by Sutton (1947, equation (6), p.429) and the substitution of the form

$$\sigma^2 = \frac{1}{2} C_z^2\, x^{2-n}$$

The pollution at Staythorpe will also include a "background" contribution, a, due to distant sources, and the total average value, p, over any octant may then be written

$$p = a + b\,\frac{f(\theta)\,S}{\bar{u}} \qquad (3)$$

where, if p and a are in units* of mg SO_3/day 100 cm^2, S is the number of tons of sulphur dioxide discharged per month in a particular season, and if $\bar{u}$ and x are expressed in knots and yards respectively,

$$b = 8.511 \times 10^3\, g(h,x) \qquad (4)$$

In equation (4), $g(h,x)$ is still in c.g.s. units.

The extent to which the relation (3) is supported by the data is examined below, firstly with regard to the functional form and secondly with regard to the absolute magnitude of the quantity $g(h,x)$.

We proceeded in the following manner:-

(a) a circle was drawn round the power station on each pollution diagram (see Figure 2) to represent a distance of 1,500 yards from the station. Radial lines were also drawn in order to indicate the octants and to divide the circle into eight equal arcs.

(b) the quantity $f(\theta)S/\bar{u}$ in respect of each octant was calculated for the three winters and three summers, an allowance being made for the percentage of calms that occurred in each season.

(c) from each pollution diagram the actual seasonal concentration in each octant at a distance of 1,500 yards from the power station was estimated.

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* For the conversion involved here a relation was supplied by the Fuel Research Station in the form: "concentration in parts per 100 million of sulphur dioxide = 4.2 x pollution in mg SO_3 /day 100 cm^2".

/Taking

Taking the seasons separately, the data formed 24 pairs of values of the quantity $f(\theta)\,S/\bar{u}$ and of the corresponding estimated actual concentration, the latter representing ρ on the left hand side of equation (3). The next stage was therefore to ascertain whether there existed between the two parameters a functional relationship of the form shown in this equation. This part of the investigation is described in the following two sections.

4. Winter pollution

The values of ρ and $f(\theta)S/\bar{u}$ that were tested for relationship are given in the following Table 4.

Table 4

Octant	1951-52		1952-53		1953-54	
	$\frac{f(\theta)S}{\bar{u}}$	ρ	$\frac{f(\theta)S}{\bar{u}}$	ρ	$\frac{f(\theta)S}{\bar{u}}$	ρ
0 - 45°	2.12×10^2	.80	13.09×10^2	.97	10.50×10^2	1.00
45 - 90°	2.58	.76	10.11	.97	17.24	.92
90 - 135°	3.74	.94	7.08	.98	15.47	1.10
135 - 180°	5.27	.89	15.27	1.04	22.43	1.28
180 - 225°	6.11	.95	28.66	1.12	31.16	1.35
225 - 270°	5.77	.97	15.94	1.10	12.56	1.32
270 - 315°	3.65	.96	8.60	1.00	6.92	1.29
315 - 360°	4.03	.88	23.84	.94	12.47	1.10

NOTE: The values of ρ (observed concentration) have been rotated through 180° in order to correspond with the wind distribution.

By the method of least squares the best straight line to fit these pairs of values of ρ and $f(\theta)S/\bar{u}$ was found to be:-

$$\rho = 0.89 + 1.16\times10^{-4}\,\frac{f(\theta)S}{\bar{u}} \qquad (5)$$

The closeness of fit may be judged from Figure 3 in which are shown the straight line given by the above equation together with a plot of all the points listed in Table 3. It will be seen that for most of the points the straight line provides a good representation. However, it seemed desirable to apply some quantitative criterion in order to estimate the extent to which the straight line does adequately represent the observations.

The most suitable index is the correlation coefficient, R, between the observed values of ρ and the corresponding values computed from equation (5) above. The value of R is given by

$$R = 1 - \Sigma\epsilon^2 / \Sigma\rho_d^2$$

where
ϵ = observed ρ - calculated ρ
ρ_d = observed ρ - $\bar{\rho}$
$\bar{\rho}$ = mean value of observed ρs

Applied to the above observations and the straight line, the value of R was found to be 0.604. In the "Handbook of Statistical Methods of Meteorology", Brooks and Carruthers give, for the conditions of this problem, a value 0.404 for the 5 per cent point of R and a value 0.515 for the one per cent point of R.

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The computed value of R , i.e., 0.604, is considerably better than the one per cent point and for the winter season, therefore, we may attribute some significance to the relationship,

$$\rho = 0.89 + 1.16 \times 10^{-4} \frac{f(\theta) S}{\bar{u}}$$

between the observed pollution at Staythorpe and a quantity based on the wind distribution and the output of sulphur dioxide at the power station.

A feature of this result is worthy of special comment. The value of the constant term (0.89) can be seen by reference to Figure 1 to approximate closely to the average pollution which existed before the power station was commissioned, i.e., when S in the above equation was zero. The fact that this constant term is so nearly equal to the original background pollution lends support to the significance of the equation.

5. Summer pollution

For the summer seasons, the values of ρ and $f(\theta) S/\bar{u}$ that were tested for relationship are given in Table 5.

Table 5

Octant	1952 $\frac{f(\theta)S}{\bar{u}}$	ρ	1953 $\frac{f(\theta)S}{\bar{u}}$	ρ	1954 $\frac{f(\theta)S}{\bar{u}}$	ρ
0 - 45°	7.15×10^2	.27	9.62×10^2	.42	13.16×10^2	.58
45 - 90°	6.39	.26	7.54	.36	6.05	.44
90 - 135°	7.78	.37	8.21	.47	6.53	.46
135 - 180°	8.29	.35	10.25	.54	18.49	.44
180 - 225°	13.37	.42	18.98	.72	31.88	.78
225 - 270°	13.89	.40	15.59	.67	26.62	.74
270 - 315°	10.85	.34	8.64	.56	18.35	.63
315 - 360°	9.59	.28	10.38	.50	21.13	.64

NOTE: The values of ρ (observed concentration) have been rotated through 180° in order to correspond with the wind distribution.

As for the winter data, the method of least squares was again used and gave the following straight line equation:-

$$\rho = 0.26 + 1.78 \times 10^{-4} \frac{f(\theta) S}{\bar{u}} \qquad (6)$$

This line is shown in Figure 4 where the points of Table 5 are also plotted and can be seen to conform reasonably well with the straight line.

The correlation coefficient R, computed in the manner described in the proceding section, was found to be 0.804, i.e., a value considerably better than the one per cent point of R (0.515) and therefore significant. The constant term (0.26) can be seen from Figure 1 to be approximately equal to the summer background pollution before the power station began to function, a result which corresponds with that found in the winter analysis.

6. The magnitude of $g(h, x)$ and associated diffusion parameters

Our analysis has so far been confined to the functional form of the relationship between pollution and the conditions of output and wind distribution. We now seek to extend the analysis in a more explicit way, by examining the absolute values of the coefficient b in equation (3). From equations (2), (3) and (4) and from the values of b contained in equations (5)

/and

and (6), we may derive values of σ, the 'standard deviation' of the vertical distribution which the cloud must possess (apart from the effect of the boundary) in order to explain the observed pollution at ground level.

We need first to assign a value to h, the effective height of the source. This is equal to the height of the chimney added to the vertical distance by which the discharged gases are raised in consequence of their initial buoyancy and upward velocity. Several formulae are available for calculating the effective height in problems of this nature and we selected the Oakridge formula which is simple to apply and which was recommended by the Beaver Committee (1954). This formula is:-

$$uz = 1{\cdot}5\, v d + 0{\cdot}4\, Q_H \qquad \text{(c.g.s. units)}$$

where u is the wind velocity, v is the velocity of efflux of the gases, d is the chimney diameter, Q_H is the heat output in calories/sec., z is the height reached by the gases measured from the mouth of the chimney, so the effective height h is equal to z plus the chimney height.

If the air temperature is T_1 °A and the gases are discharged at a temperature $(T_1 + \Delta)$ °A, then assuming the gases have the same specific heat c_p and, at T_1 °A, the same density as the air,

$$Q_H = \frac{\pi}{4} d^2 v \left(\frac{T_1}{T_1 + \Delta}\right) \rho\, c_p\, \Delta$$

hence

$$uz = Q_H \left\{ 0{\cdot}4 + \frac{6}{\pi} \frac{(T_1 + \Delta)}{T_1 \rho\, c_p\, \Delta d} \right\}$$

For Staythorpe $T_1 + \Delta$ = 419°A, the chimney diameter is 20 feet and Q_H is obtained from the coal consumption, each pound of coal burnt yielding 153 cu. ft. of gas at N.T.P. With these data combined with mean wind velocities for each season, an average effective height for the six seasons was derived. The value obtained for h was 504 feet and this was approximated to 500 feet for further calculations on the magnitude of $g(h,x)$ and thence of σ.

Comparisons between equations (3), (4) and (5) and between equations (3), (4) and (6) permit the following values to be derived for $g(h,x)$, for h = 500 ft and x = 1,500 yd:-

Winter $g(h,x)$ = 2.05×10^{-5} cm^{-1}

Summer $g(h,x)$ = 3.14×10^{-5} cm^{-1}

From Figure 5, a graph showing the variation of g with σ for h = 500 feet, the values of σ corresponding to these values $g(h,x)$ can be determined and are seen to be

Winter σ = 90 or 390 yards

Summer σ = 150 or 180 yards

It is common experience that the vertical component of turbulence is greater in the summer than in the winter months and as a general rule we should therefore expect the summer value of σ to be greater than the winter value. We therefore reject the value of 390 yards for winter and conclude that the true values of σ are approximately 90 yards for the winter and between 150 and 180 yards for the summer.

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The results for σ refer to a distance of 1,500 yards from the source and can be converted approximately to an angular deviation given by $\tan^{-1} \sigma/1500$. The values obtained are 3.5° for the winter and between 5.7° and 6.8° for the summer months. These figures appear reasonable as regards order of magnitude and can be compared with some results recently obtained by Hay and Pasquill (1956) and reported to the Meteorological Research Committee. Their experiments, which were conducted at Porton in the early autumn, included the measurement of the vertical distribution of airborne particles released mainly at a height of 500 feet and sampled at downwind distances up to 500 metres. The results indicated that the vertical spread of the cloud of particles varied approximately linearly with the distance of travel, and the corresponding standard (angular) deviations fell in the range 1.7° to 6.0°. These values are very similar to those derived above from the Staythorpe data.

Another parameter which can be evaluated is the vertical diffusion coefficient, C_z, which occurs in Sutton's treatment of diffusion and is given by

$$C_z = \sigma \left(\frac{2}{x^{2-n}} \right)^{\frac{1}{2}}$$

Taking $n = 0.25$, $x = 1,500$ yards and the winter and summer values of σ in turn and working in c.g.s. units, we find

Winter $\quad C_z = 0.37 \text{ cm}^{\frac{1}{8}} = 0.21 \text{ m}^{\frac{1}{8}}$

Summer $\quad C_z = 0.49 \text{ to } 0.75 \text{ cm}^{\frac{1}{8}} = 0.35 \text{ to } 0.42 \text{ m}^{\frac{1}{8}}$

These values for C_z may be compared with those obtained by Stewart, Gale and Crooks (1954) from measurements of the diffusion of the gases discharged from the Harwell Pile. For distances up to 800 metres they found the following mean values of C_z

Inversion conditions	$.11 \text{ m}^{\frac{1}{8}}$
Adiabatic "	$.20 \text{ m}^{\frac{1}{8}}$
Lapse "	$.25 \text{ m}^{\frac{1}{8}}$

At greater distances, up to 1,050 metres, Stewart et al report values of $0.32 \text{ m}^{\frac{1}{8}}$ for C_z in lapse and adiabatic conditions.

The present results are thus obviously consistent with two independent experimental studies of the vertical diffusion from an elevated source. Moreover, it might even be argued that the rather greater degree of vertical diffusion implied by the present results is in keeping with the fact that they represent an average over several months whereas the results with which they are compared represent averages over the order of one hour. On the other hand it should be remembered that the treatment applied here is only approximate, especially in the sense that the influence of the variation of output and wind speed on the effective height of the source are neglected. The detailed agreement obtained is therefore to be regarded as fortuitous to some extent.

7. Concluding discussion

It has been shown that for the three winter and three summer seasons that have formed the period of this investigation, the distribution of sulphur dioxide pollution around Staythorpe was significantly related to a parameter incorporating in a simple fashion the sulphur emission at the power station and the distribution of wind speed and direction near the site. In addition, the relationship found has provided values of diffusion parameters which compare reasonably with the values obtained experimentally by other workers.

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In the present analysis we have not attempted to examine the separate effects of output and wind conditions. However, from an inspection of Figures 3 and 4, bearing in mind the variation in output from year to year (see Table 1), it is fairly evident that the relation between χ and $f(\theta)/\bar{u}$ alone would be less systematic than that between χ and $f(\theta)S/\bar{u}$.

This investigation, which has been based on a detailed study of the pattern of pollution around a power station, has led to conclusions which could not have been drawn from a consideration of the average pollution in the area. Our results therefore suggest that the average pollution, which is commonly used in studies of this kind and is in fact the unweighted mean of readings taken at various distances from a power station, may prove misleading both for year by year comparisons at one power station and for comparisons between different power stations. Consideration might well be given to devising a system of weighting the individual observations in accordance with the wind distribution and the distance from the source before local averages are worked out.

8. Acknowledgement

We are glad to acknowledge the assistance of the Central Electricity Authority and of the Fuel Research Station for providing much of the data used in this investigation.

References

1. Meetham, A.R. Atmospheric Pollution (Pergamon Press, 1956) p.167.

2. Sutton, O.G. 1947 Quart J. R. Met. Soc., London, 73, pp.426-436.

3. Final Report of Government Committee on Air Pollution. Cmd.9322 (H.M. Stationery Office, 1954).

4. Hay, J.S. and Pasquill, F. 1956 Measurements of the short range diffusion of airborne particles at a height of a few hundred feet in the atmosphere. Meteorological Research Papers, Air Ministry, London, No. 1006.

5. Stewart, N.G., Gale, H.J. and Crooks, R.N. 1954 The atmospheric diffusion of gases discharged from the chimney of the Harwell Pile (BEPO). A.E.R.E. Report HP/R.1452.

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STAYTHORPE POWER STATION

Seasonal Coal Consumption and Pollution Averages

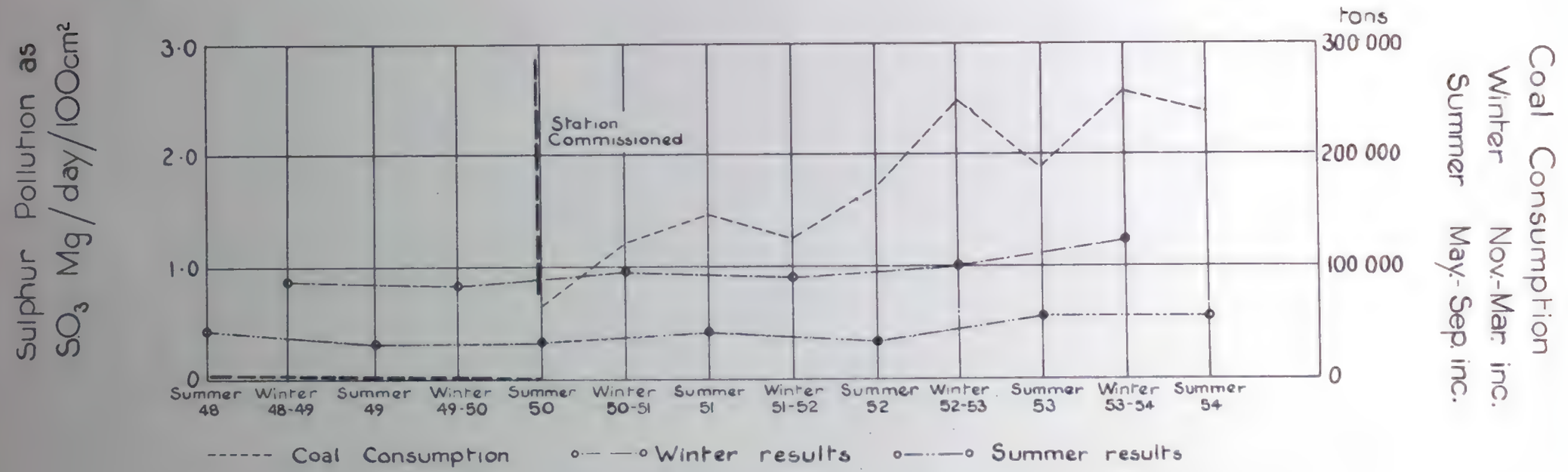

Figure 1.

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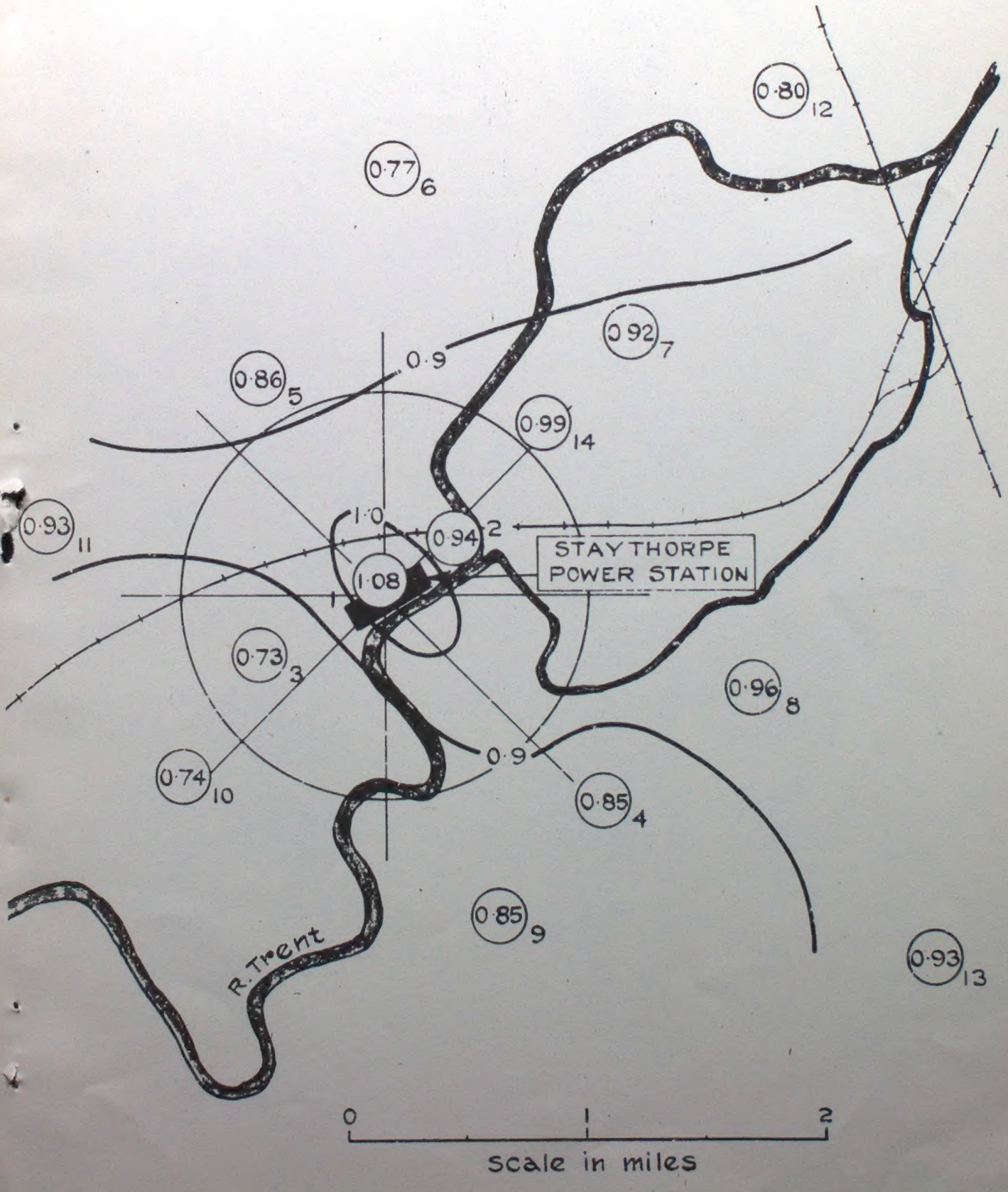
STAYTHORPE

Sulphur Dioxide as mg. SO_3/day/100 cm.^{2}

WINTER

Nov. 1951 – MAR. 1952

FIG. 2.



STAYTHORPE

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Winter Seasons

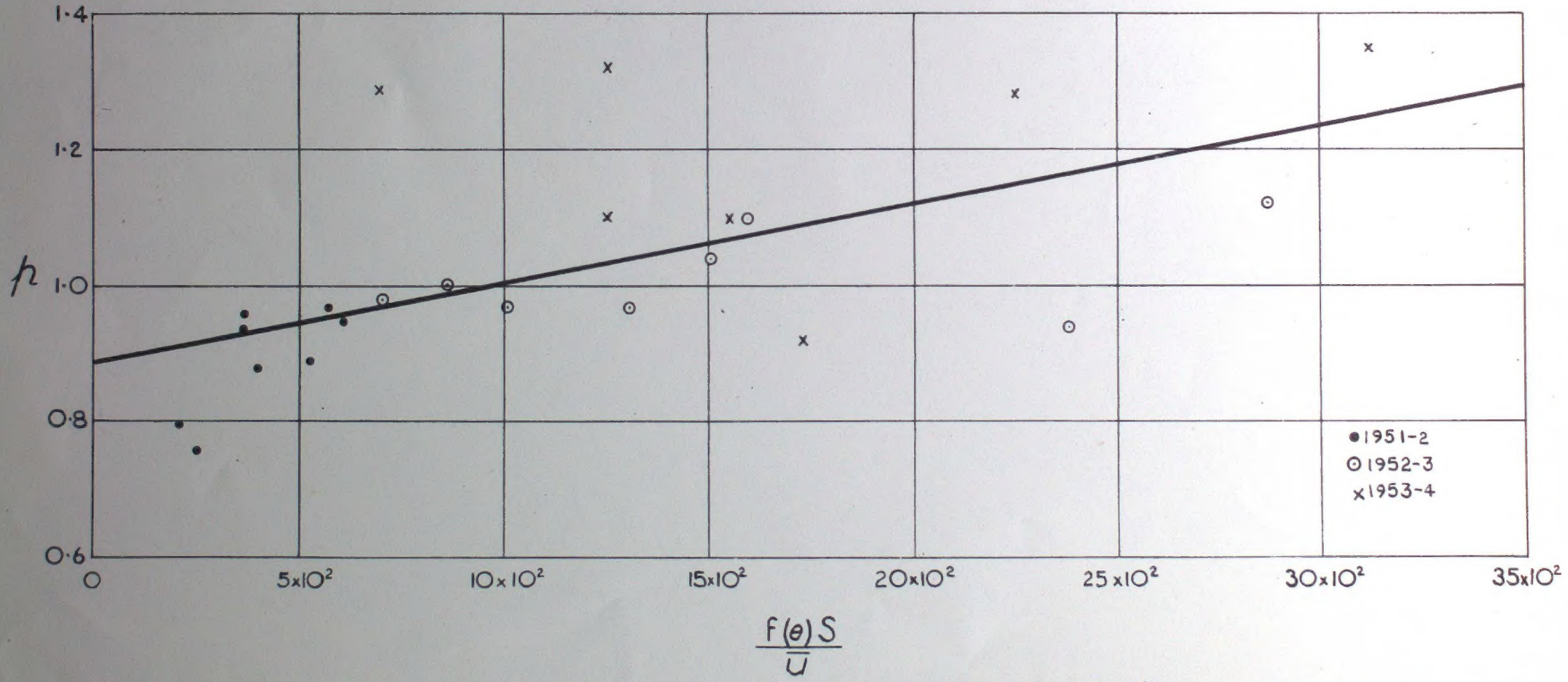

Figure 3 Relation between Observed Pollution (h) and $f(\theta)S/\bar{u}$ for Winter Seasons
Equation to Straight Line $h = 0{\cdot}89 + 1{\cdot}16 \times 10^{-4} \frac{f(\theta)S}{\bar{u}}$

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STAYTHORPE

Summer Seasons

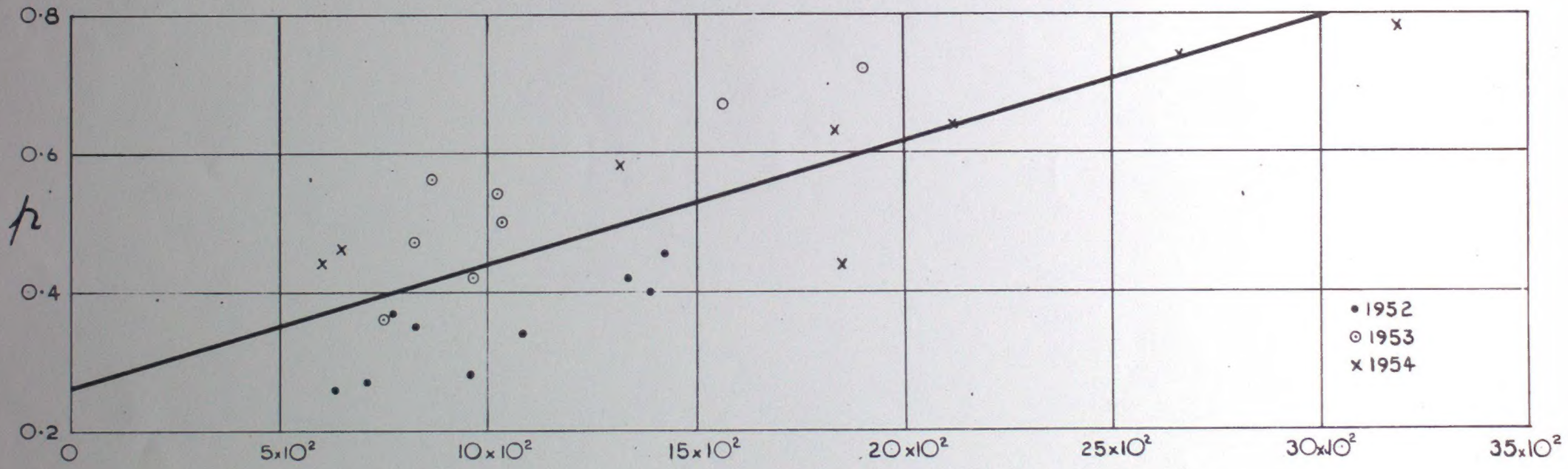

Figure 4. Relation between Observed Pollution (h) and $f(\theta)S/\bar{u}$ for Summer Seasons.
Equation to Straight Line $h = 0{\cdot}26 + 1{\cdot}784 \times 10^{-4} \frac{f(\theta)S}{\bar{u}}$

$$g = \frac{2}{\sqrt{2\pi}\,\sigma} e^{-\frac{h^2}{2\sigma^2}} \text{ in c.g.s. units}$$

$$g = \frac{8{\cdot}75 \times 10^{-3}}{\sigma} e^{-\frac{h^2}{18\sigma^2}}$$

h in feet
σ in yards

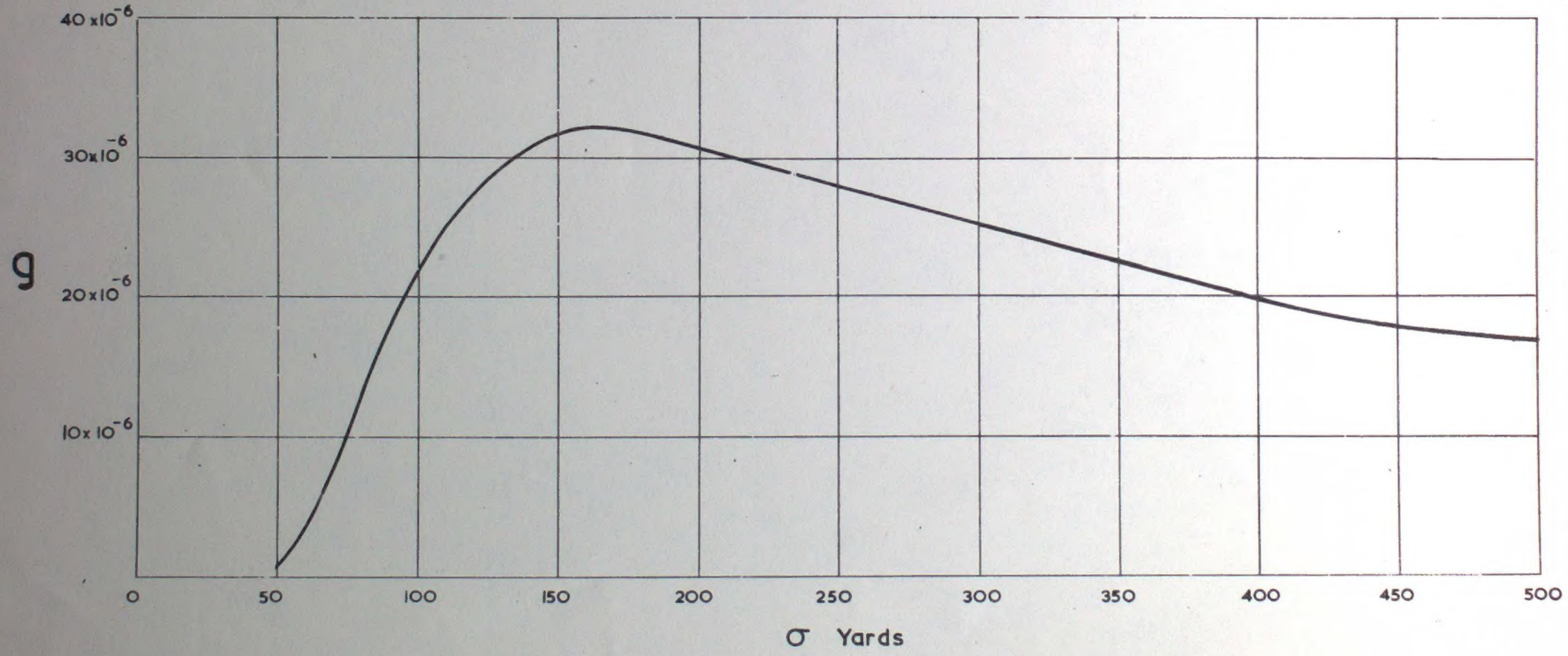

FIGURE 5 Variation of g with standard deviation, σ for h = 500 feet.